

**AN AESTHETIC
REPRESENTATION OF ALFRED
NORTH WHITEHEAD'S
PHILOSOPHY OF
MATHEMATICS EDUCATION**

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A STORY

Ali's in the World of Heavens

Ali was beginning to tire of sitting by his brother on the bank now that he had finished his mathematics homework. He had excelled at today's work to such a degree that he could do the problems without even thinking about them.¹ Deeply bored, Ali peeked at the mathematics book his brother was reading. The book contained no numbers; Ali only saw meaningless symbols.² "What is the use of a mathematics book", Ali thought, "without numbers or practical problems to solve?" But as he studied the book, Ali realized the symbols were not stable. Instead, as his brother read, the symbols were mysteriously changing and transforming into a different set of symbols. Ali's brother did not seem to notice the changing symbols for it was his thoughts that were defining the meaning of the symbols. Ali imagined his brother's brain expanding, getting bigger and bigger as it was filled with all the things he has been reading.

The hot sun made Ali feel very sleepy and stupid, and he turned his attention from the perplexing book to the small pleasure of making a daisy chain. Ali wondered if it would be possible to make a chain with an infinite radius and whether it would be worth the trouble of getting up and picking an infinite number of daisies. He also wondered whether the diameter consisting of all the real numbers would be larger than the total number of daisies in the circumference?³

Pondering this question, Ali was saddened to realize that no daisy could live long enough to finish this infinite chain; it would either shrivel and die or be brutally consumed by a rabbit - a rabbit like the one with a white head and pink eyes that had just run by him. This idea of the daisies and a selfish rabbit seemed nothing more than sad⁴ until Ali heard the white-headed rabbit (or just Whitehead for the sake of simplicity) exclaim, "Oh dear! Oh dear! Not again! Why do I have to lose my ability to talk every time I am here?"⁵ This was no ordinary rabbit.

Ali wondered whether Whitehead was real at all,⁶ or whether he might have imagined this impossible creature. "I think I should simply ignore it and continue to act according to my interest⁷, which is to make this infinitely large chain of daisies," Ali thought. Then, something even stranger happened.

The rabbit did not appear to be speaking because his lips weren't moving, but Ali could still hear Whitehead saying, "Importance is derived from the immanence of infinitude in the finite".⁸ Ali was confused about how to respond - whether in words or through his thoughts - since Whitehead seemed to magically communicate through thought rather than speech. Finally, Ali made up his mind. Rather than putting his thoughts into words, he simply thought, "To make a daisy chain is my desire, which is a matter of fact that is so important to me that I should not waste my time talking to you". Despite his desire to build an infinitely large chain of daisies, and its relative importance to him, Ali's curiosity burned. He wondered why the rabbit was complaining that it can't talk if it didn't need to talk? Once again, Ali heard the rabbit's voice in his head, "Expression is

founded on the finite occasion⁹ and only animals express themselves unconsciously. I am not an animal, so stop referring to me as *it!*” Ali was stunned upon this outburst. This was such an unusual occurrence in an ordinary day. Was it really an ordinary day?

At this point, Ali found himself thinking about the book that his brother was reading, and he wondered about the mode of communication between his brother and the author of the book. Were the numbers elements of a finite world; were the symbols elements of an infinite world? Perhaps the relationship between numbers and mathematical symbols paralleled the relationship between language and thought. Perhaps these concepts were related to each other just as any reflection is related to the original object. It is never a perfect translation. There is always some distortion. Which element is more important? Which element reflects the infinite?

Ali shouted angrily, “Who are you? What are you?”

Through his thoughts Whitehead explained, “I was once a nebula and later united with some dust particles from the Moon. Finally I arrived on Earth in the form of a white-headed rabbit”.¹⁰ Without thinking another word, Whitehead started running. He was running purposefully to reach a destination rather than trying to escape from Ali. He seemed to know that Ali was going to follow him wherever he went, even down a dark, unmarked hole in the ground. Following his curiosity and the intriguing rabbit, Ali found himself in a new alternate reality.

Down, down, down he tumbled. Suddenly, Ali landed on a heap of sticks and dry leaves with a loud thump! Looking around, he saw he had landed next to a man in a suit. With a twitch of his nose, the man introduced himself saying, “Hello, I am Whitehead. I’m glad you have arrived in the World of Heavens. This passage between our worlds is called Gabriel’s Horn.¹¹ We tried to make the passage nicer, but we could never finish painting the inside since it is infinitely large. The hole is too small and cumbersome for two people at once. I’m doing entirely too much talking! This is what you need to know. This world is very different from your world. Here, everyone and everything can talk. Take care, as all your thoughts are automatically translated into words that others will hear.¹² Like raindrops on a pond, your words will constantly fall and the ripples of consequence they create may be hard to control. Welcome, and good luck”. At this, the man ran away, leaving Ali alone in his quest for eternal knowledge.

With Whitehead’s warning weighing on his mind, Ali wandered around a bit, trying to figure out what this whole new world was about. After a bit, he came across someone who was shaped like a symbol. This person looked exactly like the letter “U” and “U” was wearing a black tie. He or she - it was hard to know - introduced itself in the fashion of James Bond saying, “Hello. I am Stan - Under Stan. I’ll be your guide in this world, and I shall do my best to help you understand everything as it was, as it is, and as it shall be”.

Hearing the word “everything” was enough to scare Ali. He knew he would never have enough time to understand all the weird things in this place, and he was already yearning to return home and feed his cat. “Forget

about your cat. Let's concentrate on the daisies," U said. U continued excitedly, "Wasn't it you who wanted to have an infinitely large chain of daisies?" U asked the question in a very Aristotelian way, so Ali nodded his head in agreement. "What would you need for such a chain besides infinitely many daisies?" U continued.

"What a stupid question!" Ali thought, but he tried to dismiss this idea so as not to offend U. Instead he responded, "I would need time, an infinite amount of time". U was happy to hear this answer and calmly said, "Well, then my Master shall give you what you wish for since he has infinite patience".¹³ U pointed out a huge green sign with the following words written on it:

I am Master of this World;

*And what I know not, is not knowledge.*¹⁴

Ali rebelled against this idea, once again falling back on the constraints of time and place found in his world, rather than exploring the logic of this new world. Ali wondered how this statement could make any sense when even human knowledge was constrained by time and place. Moreover, Ali had his own experiences and knowledge to consider and perhaps that would be outside the master's domain. Directing his thoughts to U, Ali asked "What about my own perspective? What about my imagination?"

U had not been chosen to initiate newcomers without reason, and he had been in this business for years. This was perhaps the thousandth time that he had been forced to answer such an arrogant question. However, instead of getting angry he chose to stay calm and intelligently answered, "You are forgetting the fact that our Master has infinite knowledge. Your imagination is actually part of His imagination. He is giving you what you need in a non-changing world so you can grasp the meaning of everything, and perhaps even build your chain of daisies."¹⁵ His gift is infinite time. Now that you have infinite time, and because the daisies will not die, you may have a chance to succeed in your quest. You may also be interested to know we have some rational daisies, as well as irrational ones. They are all real, though. You can build a chain in which the diameter can be smaller or greater than the circumference. It is all up to you".

Ali wondered how this was possible. How could one's knowledge not be limited by the present parameters of the universe?¹⁶ Ali could only imagine an answer if he believed that the master had existed in both realities since the beginning of time. But that would also mean Ali was going to be forced to stay here with the master for the rest of time in order to understand him. Ali was upset, thinking he would not be able to see his brother anymore. "Perhaps," he wondered, "the best way to proceed is to just try and enjoy the present, exactly as it is right now". Ali maintained hope that he would be able to change the future through his actions in the present and hopefully return back to his own world. However, the question remained. Were these two worlds somehow consistent with each other, or were they contradictory?

U told Ali to follow him and it showed him an apple. Admiring the apple, Ali noticed how wonderful this apple was, how perfect and delicious. Ali also remembered the rotten apple that he had given to his cat the day before.

That is, he tried to remember the rotten apple. He pushed himself, trying to visualize the apple, but he just couldn't imagine the rotten apple, because the image of the perfect apple defeated all the other thoughts in his mind. "How could this happen?" Ali asked.

"Nothing ever changes in this place, neither for better or worse, even in my mind," U replied in a philosophical manner. It added that, "Our universe escapes from the limitations of the finite. Things that you find contradictory in your own world are simply consistent in this world. This is because our Master can presently see the past and future,¹⁷ and any contradictory things work in harmony in our world". U tried to explain this unexpected reality as the two walked along a long path through a garden. Like everything here, the garden was unusual, filled with animals and flowers chatting with each other. Not only did the animals and plants seem to be communicating, the stones, and even the dust particles floating in the air seemed to be communicating. Ali even saw the number π in the air flirting with a circle.

"This is too chaotic. I can't understand a word when so many things are talking at once," Ali complained. He was burning to know more about this world that was pulsating with consciousness, and wished he could interview each element of the world to learn about its experiences. During his journey, Ali met with a wide variety of things, and each seemed to be living and talking. They were all working in harmony, without conflict. Ali saw a tree producing apples and serving them in a respectful way to others. As its presence thrived in the environment, Ali heard the tree saying, "Thank you for bringing me into existence, Master. You are beautiful and perfect; you give life to me without expecting anything back but my appreciation". If only he had the time, Ali would ask the trees how photosynthesis felt inside their bodies, and what they felt when chloroplasts transformed solar energy into ATP. He longed to talk to the number π and learn about the beautiful circles around him. Everything here seemed to be beautifully connected.

Sadly, there was no time to explore. Instead, someone called Anaxagoras approached Ali and told of his adventures trying to square the circles.¹⁸ Ali wondered if "squaring" was slang only when the guy called Tangent said he used to live next to the most beautiful circle in the world, but could never get inside its heart.

Ali explored the World of Heavens, never feeling as though he had enough time. For days on end he observed the elements of the land living in harmony witnessing their contributions to the environment, and trying to learn about the experiences of each element.¹⁹ At the end of each day, Whitehead visited Ali and tried to explain the ways of this world, naming all the elements Ali had encountered. Whitehead only introduced the name of something after Ali had observed it for himself. When Ali asked why Whitehead wouldn't tell him the name of things before encountering them, Whitehead explained, "In the World of Heavens, we must first let you see everything before you learn its name. In fact, you were supposed to see these things in the other world and then learn the names here, but for some reason you were rushed and born prematurely into this world".

Grappling with learning so many new concepts, Ali felt as though he was reading a book in which the chapters included everything ever written by the

master - this master who claimed to have knowledge of all unchanging things, and represented them in the book with their true value and meaning. After spending time in this place, Ali found himself thinking, “This should be the true world, a world which is both complete and consistent.” Ali threw himself into learning the ways of the new world so that he could gain an intuitive understanding of how this foreign world operated. Ali learned about tangents, and circles; he learned about pi. He even had homework and tests in which the answers always referenced the lessons he had learned that day. There were never any trick questions²⁰ and by the end, Ali was able to answer many questions without much thinking.²¹

Suddenly, Ali awoke from his long sleep to find his brother shaking him. Ali, understanding he had returned to his native world, saw that with his visit to the World of Heavens his present world had changed because his views and interpretation of this changing world had changed. Ali asked to see his brother's book and studied it, looking for clues that might show him how to understand the universe from a new point of view that would combine all three perspectives: the book's perspective, the universe's perspective, and Ali's own perspective. Ali found that although the book, Ali, and the universe appeared to represent three different perspectives, they were basically telling the same truth. He discovered that as he knew more about himself, he understood the book better, and as he better understood the book, he better understood the universe. Thus, Ali realized that he himself was a small example of the entire universe. From his experience in the World of Heavens, Ali had learned infinity was not possible in this world. All the forms of this world were essentially referent beyond themselves, even the numbers and symbols found in mathematics. Ali now understood that in this world he would learn and grow until he was ready to reenter the World of Heavens and its harmonious, coherent existence. Ali had come to see the trees on the bank as serving apples to people, and being replaced by new trees when the time came. Ali now understood that consciousness infused every particle of dust, every breath of life, and every concept in the universe. Ali saw the master as alive and charged with supervising and controlling the universe in order to prepare all his subjects for the unchanging world of eternity.

As for the rabbit, Whitehead was in the fields looking for another human to take to the World of Heavens for a taste of eternity.

CONCLUSION

By passing through the Gabriel's Horn, Ali learns that every form in this world refers to a realized being in the World of Heavens. Meanwhile, Ali's brother studies his book to learn more about this world. It is up to the reader to decide which character is more successful in reaching general truths by starting from particulars. In this story, Ali's brother represents the philosopher Leibniz, who aimed to inherit two thousand years of human thought (Whitehead, 1938). Ali represented William James, who "intuitively discovered the great truth" (Whitehead, 1938, p.4). The most important lesson Ali learned from his experience in the World of Heavens was the true, organic, and connected state of all things. Before visiting the World of Heavens, Ali was saddened at the idea of organisms such as animals, vegetables, even daisies dying disconnected from one another in their own worlds. After learning the truths of the World of Heavens and seeing that everything is dependent on one another to survive, he came to the romantic view that the everything is conscious and lives in harmonious connection with one another. The apple tree was serving its apples to other organisms, and doing it because the goal of its existence is to provide abundantly and thrive, rather than out of any sense of obligation. Death made way for others to live and experience life. It wasn't a cause for sadness. Ali was illuminated by the understanding that the opportunity to live was reason enough for happiness. The rabbit presented this lesson to Ali, representing Alfred North Whitehead's belief that there was only one subject worth studying and that was life in all its manifestations (Whitehead, 1929).

POSTFACE

Ali's in the World of Heavens is an aesthetic learning experience that depicts Alfred North Whitehead's philosophy of education as it was described in his books (Whitehead, 1910; 1911; 1929; 1934; 1938a; 1938b; 1938c; 1938d; 1985) and by other analyses (Ernest, 2000). Process philosophy implicitly guides much of the discussion throughout this paper. Many of the concepts are explained in the endnotes to help readers see hypothesized connections. In short, this narrative uses a familiar style to explain *becoming* as it belongs to the temporal world, compared to *being*, which is characterized by an eternal non-changing world. This short story demonstrates how all *becomings* are moving to a destination to fulfill their potential - thus becoming *important beings* rather than remaining simply *expressions*.

ABSTRACT

This short story addresses our quest for knowledge from the perspectives of two brothers. One brother, Ali, follows the educational system proposed by Alfred North Whitehead, while the other brother educates himself through the classical method. The story is based on Lewis Carroll's *Alice in Wonderland*, and begins with a romantic episode in which Ali expresses his desire to know more about infinity. At the same time his brother is reading a book - the content of which seems to change depending on the reader's interpretation. While his brother struggles to understand a book that is more than a collection of inert ideas, Ali is guided by a white-headed rabbit into a Gabriel's Horn passage. Here he visits an alternate reality where he learns about the true reality of life and consciousness. When he returns to his brother, Ali's perspective is forever changed, and he must synthesize the lessons of this strange world in a life that is now extraordinary.

Keywords: Alfred North Whitehead, process philosophy, mathematics education.

REFERENCES

- Dauben, J. W. (1979). *Georg Cantor: His mathematics and philosophy of the infinite*. Cambridge, Mass.: Harvard University Press.
- Ernest, P. (2000) Whitehead and the implications of the process metaphor for mathematics. *Interchange*. 31, 2-3, 225-241.
- McMahon, D. L. (1999). Alfred North Whitehead. In Boston University (Ed.), *Boston collaborative encyclopedia of western theology*. Retrieved March 30, 2010, from <http://people.bu.edu/wwildman/bce/whitehead.htm>
- Ocken, S. (2007). Reflections on the NCTM focal points. *New York City HOLD National*. Retrieved from <http://nychold.com/art-ocken-07a.html>
- Whitehead, A. N., & Russell, B. (1910). *Principia mathematica*. Retrieved from <http://name.umdl.umich.edu/aat3201.0001.001>
- Whitehead, A. N. (1911). *An introduction to mathematics*. London: Williams & Northgate.
- Whitehead, A. N. (1929). *Aims of education & other essays*. New York: Macmillan.
- Whitehead, A. N. (1934). *Nature and Life*. Chicago: University of Chicago Press.
- Whitehead, A. N. (1938a). Lecture one: Importance. In *Modes of thought* (pp. 1-27). NY: Macmillan.
- Whitehead, A. N. (1938b). Lecture two: Expression. In *Modes of thought*. (pp. 28-57) NY: Macmillan.
- Whitehead, A. N. (1938c). Lecture three: Understanding. In *Modes of thought* (pp. 58-87). NY: Macmillan.
- Whitehead, A. N. (1938d). Lecture six: Civilized universe. In *Modes of thought* (pp. 143-181). NY: Macmillan.
- Whitehead, A. N. (1985). *Process and Reality* (Corrected ed.). (D. R. Griffin & D. W. Sherburne, Eds.). New York: The Free Press.

FOOTNOTES

1 A. N. Whitehead believes education should achieve automaticity so that the mind is free to study higher-level problems (Ocken, 2007). He disagreed with the idea that people must always think about what they are doing. He believed thought is useful only at decisive moments (Whitehead, 1911), and it should come with a decisive consciousness, which results from concentration and elimination of the irrelevant (Whitehead, 1938d).

2 The book, with no numbers and no practical problems, is a reference to Alfred Whitehead's unsuccessful effort with Bernard Russell to reduce mathematics to logic in *Principia Mathematica* (1910). Despite all their effort to put certainty back into the foundations of mathematics, the book was a failure in creating a closed system of truth. Nevertheless, it turned out to be an open-ended starting point for Alfred N. Whitehead (Ernest, 2000), as well as for Ali. However, Ali was the one who discovered that the symbols were changing themselves, as they were causing a change in his brother's mind. The brother was unaware of that change, and for him the book was only a source to understand the static matter-of-fact world around him.

3 Both real numbers and natural numbers are infinitely large, as well as the daisies and the real numbers in the number plane. Cantor showed that the real numbers' infinity is larger than the infinity of natural numbers. Interested readers may want to read Dauben (1979) who opposed that actual infinity is an expression of any sort of reality.

4 A. N. Whitehead's process philosophy considers life as an organic entity. The reality (and learning) is beauty-centered and holistic (Ernest, 2000). At this point, Ali sees the world as composed of disconnected bodies that are in constant competition with each other.

5 Rabbit's inability to talk explains why A.N. Whitehead thinks failure of language is the great problem of philosophy in the finite world (1938c). This relates to our inability to express concepts of the infinite world (where important things come from) by using the tools of the finite world (language). Secondly, the rabbit's inability to talk metaphorically represents how A.N. Whitehead separates humans and animals from vegetation; yet he still emphasizes that the main characteristic of the animals is their lack of having the same level of expression as humans.

The Whitehead in the story went through a Kafka-style metamorphosis in Ali's world and became a rabbit. Therefore, he doesn't possess the ability to express himself as humans do. The only communication tool that remained was what he used to have in his original world - speaking without uttering words.

6 Understandably, Ali doubts the reality of the rabbit as he has not met a talking rabbit before. Utilizing such a knowledge that is not connected to past experiences is a difficult task for Ali. A. N. Whitehead formulates this by saying, "each actual occasion is a distillation of the totality of the past (McMahon, 1999, chap. 3)".

7 A. N. Whitehead (1938a) explains the notion of importance as the "interest, involving [the] intensity of individual feeling which leads to publicity of expression" (p. 11). Ali's interest in the story is to build a daisy chain, which is a definitely the way he chose to express himself. However, this interest becomes important for him to such a degree that he ignores all the other things around him, including a talking rabbit.

According to A. N. Whitehead, importance should be based on the unity of the universe, whereas the interests stem from the individuality of the details. As interest always modifies expression, Ali is in need of guidance that would emphasize higher order thinking skills — connected to his individual interests— that will lead him to the unity.

Importance based on the unity of the universe comes from infinity and learning becomes important as it leads to generalizable facts. Expression is displayed in the temporal and finite world which is shaped by individual interest. A. N. Whitehead's education connects the finite with the infinite, and aims to bring importance and interest together in the classroom.

8 (Whitehead, 1938b, p.28)

9 (Whitehead, 1938b, p.28)

10 This reflects the organic connectedness of everything in the changing temporal world. According to A. N. Whitehead (1938a, p 30) "human body is that region of the

world which is the primary field of human expression”, thus the correct answer to *who* or *what we are* should be in whatever way we express ourselves. Because our body and mind are in constant change, each of the atoms within our body has been, and will be, a part of something else.

11 Evangelista Torricelli is the inventor of Gabriel's Horn figure which has infinite surface area, but finite volume. The name refers to the Archangel Gabriel who blows the horn to announce Judgment Day. In the story, it is the metaphorical connection between the finite and the infinite worlds.

12A. N. Whitehead claims “language is thought, and that thought is language,” (1938b) and it is the first step to achieve in the precision stage of a child's cyclic growth after the initial romance period. Ali's interest to build an infinitely large daisy chain was the first step in his romance period, where his interests were shaped. At this stage, Ali is about to enter the first precision stage, and he needs to know the perfect method to communicate with the elements of the World of Heavens in order to reach the stage of generalization. Such a language doesn't exist in the temporal world, but the ripples of his words with this perfect language will help him reach the final stage.

13 (Whitehead, 1985, p. 346).

14 This “...is a rhyme which fits onto the tradition respecting Dr. Whewell, who was Master of Trinity College, Cambridge, about eighty years ago. The rhyme is well-known, and runs thus:

I am Master of this College;
And what I know not, is not knowledge.

This attitude is always prevalent in the learned world. It sterilizes imaginative thought, and thereby blocks progress” (Whitehead, 1938c, p.59).

By providing this example, Whitehead opposes the God-role played by the teachers, who see themselves as the owner of eternal knowledge. According to A. N. Whitehead, it is not possible for a teacher to know everything that is necessary for pupils. Students need to learn not only specialized knowledge of the teacher, but also how it is connected to life and all other subjects. A. N. Whitehead's (1929, p.4) definition of education is, “...the acquisition of the art of the utilization of knowledge”. As his definition suggests, the role of the teacher is to teach how to learn this art.

15 As this is not the real world, no such teacher exists in the temporal world. I am in no way implying that the teachers should play the role of God as the source of knowledge. I assume the sarcasm in these sentences is clear, as it targets those teachers who are very much fond of such a role.

16 (Whitehead, 1938c, p. 60)

17 A. N. Whitehead (1929) claims, “[the] present contains all that there is. It is the holy ground; for it is the past, and it is the future”. According to him, this is the type of knowledge we should be providing to our children, instead of teaching depreciation of the present or the knowledge of the past, which doesn't equip them for the present. For example, when we teach about history of mathematics we are interested in bringing attention to the experiences of the past mathematicians and showing their life so that students can relate those experiences to their own lives, and learn from the process. A. N. Whitehead believes any other type of knowledge of the past is a deadly harm to the minds of our students.

18 Squaring the circle is the historical challenge of constructing a square with the same area as a given circle by using only a finite number of steps with compass and ruler. Informally, one can also claim that Ali's job of making a daisy chain with an infinite radius is squaring the circle and logically not possible. On the other hand, Anaxagoras was an ancient Anatolian philosopher who is known for his work on squaring the circle. It was much later proven that this task is not possible because the number pi was proved to not be an algebraic irrational number.

19 Ernest (2000) describes A. N. Whitehead's notion of organic connectedness of all things as an inspiration to ecological philosophers which “posits human knowing and human culture as an open-ended phenomenon” (p. 226). According to A. N. Whitehead, various forms of knowledge are linked, and the pupil should be thrown into every possible

combination. The student doesn't need to be taught too many subjects; but what is taught should be taught thoroughly. The most important subject matter of knowledge should be life itself (Whitehead, 1929).

20 A. N. Whitehead believes that uniform external examination is a deadly practice because there will always be some issues that are not covered by the individual teacher of the pupil, which will mean tricking the student. On the contrary, the educational system proposed by A N. Whitehead depends on the character of the students and the genius of the teacher to teach them how to learn about life (Whitehead, 1929)..

He also opposes a uniform curriculum filled with inert knowledge because educators deal with the human mind, not with dead matter. The human mind is not an instrument that needs to be sharpened by learning inert ideas and skills. Humans are social beings, and each of them is a part of a culture; you cannot postpone life until you have sharpened their minds (Whitehead, 1929)..

21 As Ali gained the automaticity of basic knowledge and gained an understanding of the well-connectedness of life around him, it is now time for him to go back to his own world. At this time, he completed the precision stage, and is ready to apply his knowledge to further generalizations in his own world.

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